Maths Fluency

…and why it's important

June 2024

What does fluency in maths mean?

Developing number sense and choosing the most appropriate method for the task at hand.

When children are fluent they are able to:

- calculate **accurately** and **efficiently**
- be **flexible** in their choice of strategies.

They feel:

- **confident** in working with numbers
- can **explain** their thinking
- can **apply** their understanding in **different contexts**.

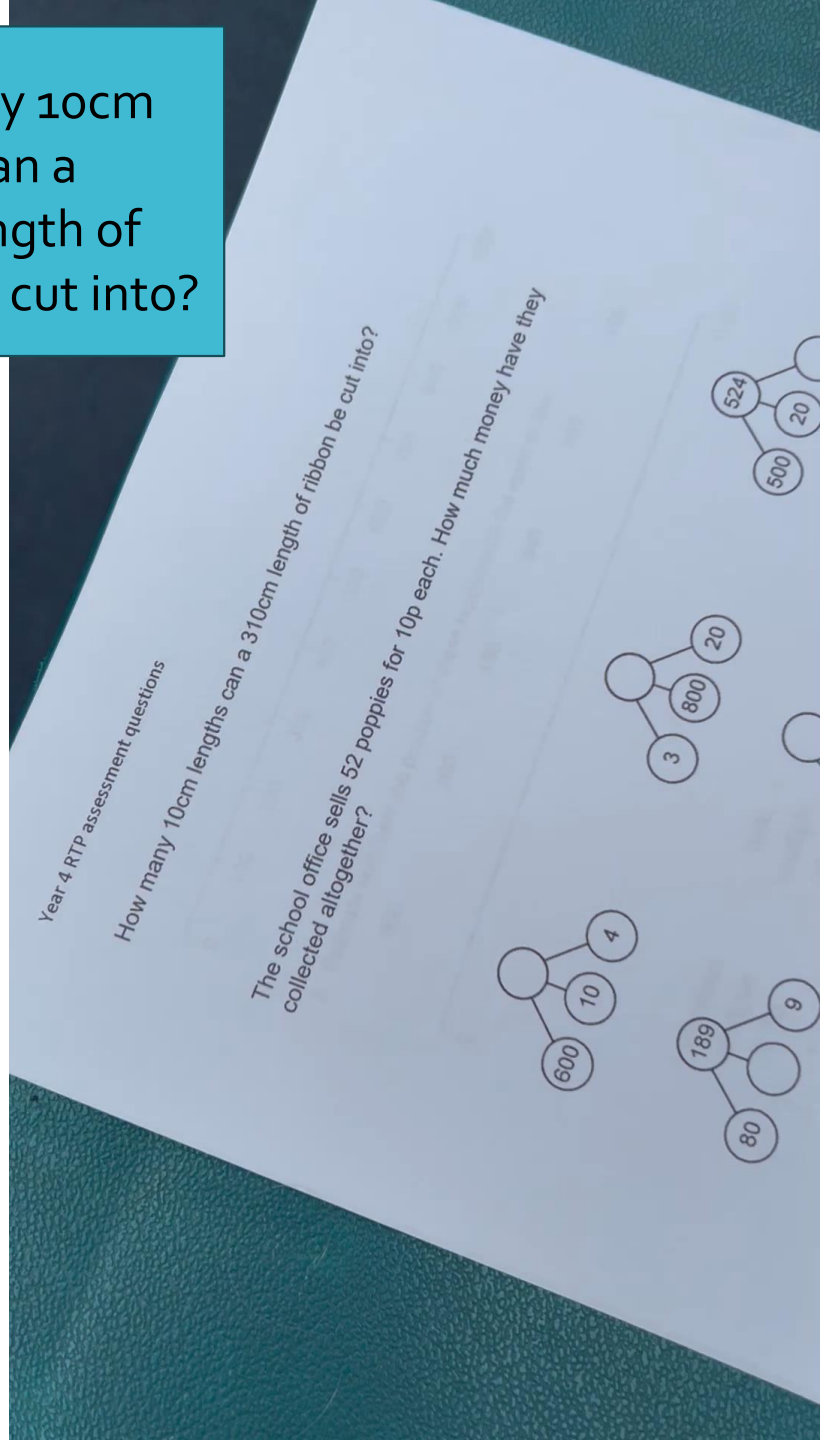
Why is maths fluency important? *

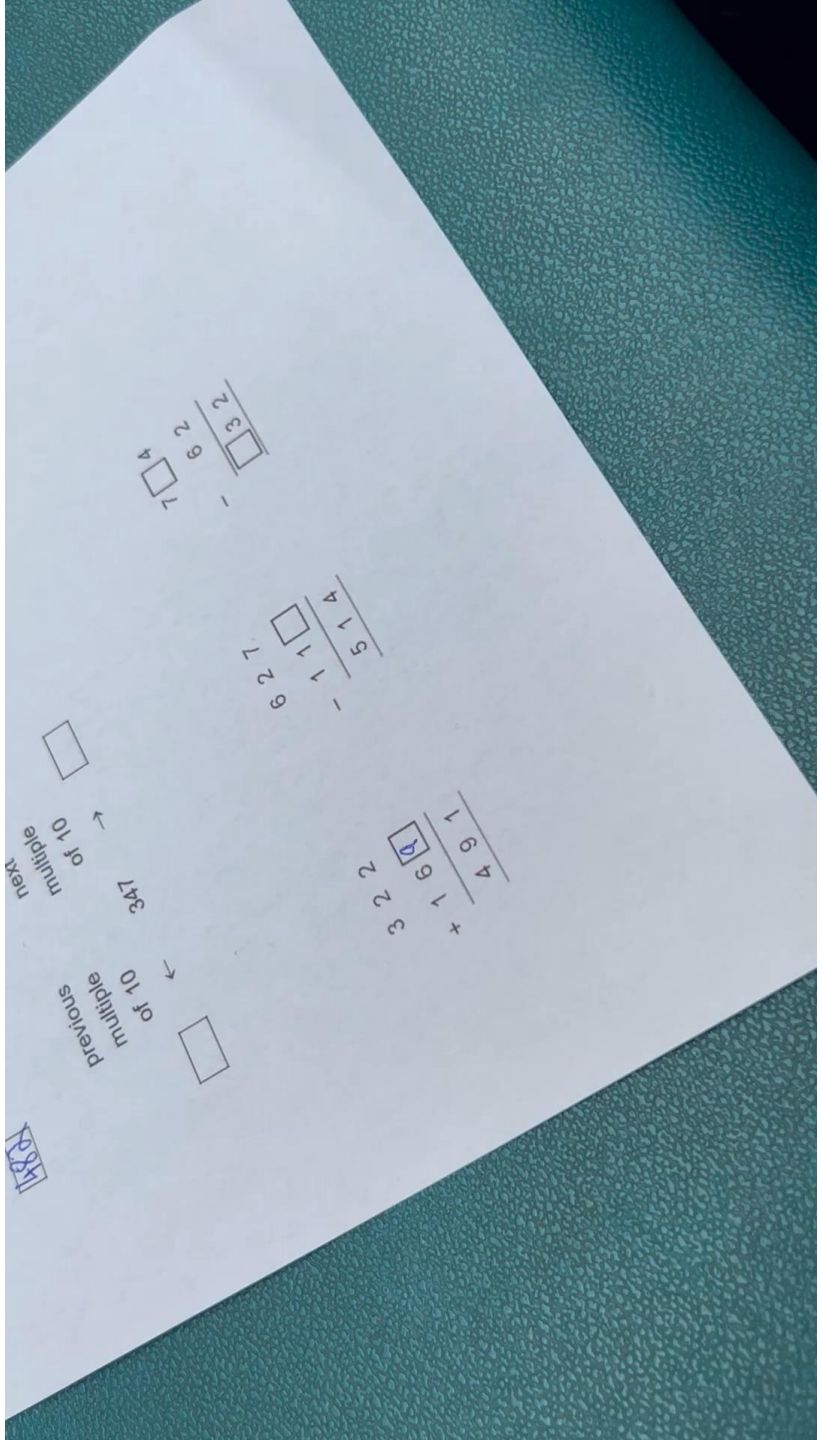
Children who are fluent have:

- the **flexibility** to select **efficient** strategies depending on the question and the context
- they are more likely to be **accurate**
- they have more time and energy to actually solve the problems
- they can tackle more complex problems with greater **confidence**

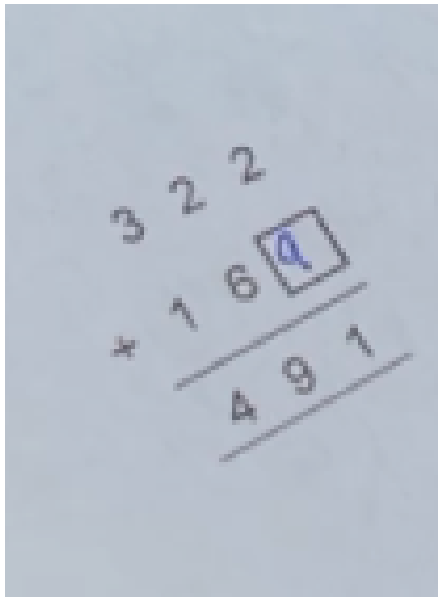
When children are not fluent, they can spend a lot of time and effort working through each small step of a calculation or problem and are likely to make mistakes.

How many 10cm lengths can a 310cm length of ribbon be cut into?





2 + ? = 1
Reasoned that he needed to add something to make 11.
Needed to use fingers to completed the calculation.



Why is maths fluency important? *

Ofsted soundbites…

sharply focused on the most important content that pupils need to master. The school has identified that pupils are not routinely given sufficient time to practise key skills such as reasoning and problem-solving in mathematics. In addition, staff do not yet consistently check that pupils have remembered the essential knowledge and skills, which means that pupils sometimes have gaps in their learning that are

their existing knowledge and skills with confidence. We scrutinised a range of pupils mathematics books. This scrutiny showed that pupils are regularly given opportunities to develop their calculation fluency and apply their mathematical understanding to solve problems and explain their reasoning. You were also asked

How do you build maths fluency?

To build a deep understanding of number and number relationships and develop fluency in addition, subtraction, multiplication and division facts and concepts you need:

- frequent practise (separate to the daily maths lesson),
- high quality mathematical talk
- use of key representations and concrete resources

What is an example of maths fluency?

Children who are fluent look for efficient methods to solve problems:

- 3 + 9 + 7 → combine the 3 and 7 first to equal 10 and then add 9 to total 19.
- 6 + 7 → double 6 is 12 so 6 + 7 must be 13 because it is one more.

These strategies would build on a child's understanding of addition being commutative and make use of the number facts they already know.

How do we teach it?

(the important bit)

The Number Facts Fluency Programme

Start of Key Stage 1 – December of Year 3

(after this – move to Times Table structured teaching programme until MTC and beyond if needed)

Modelled on the phonics programme used in early reading, the programme teaches a core set of number facts (grid facts) and teaches them systematically alongside the calculation strategies that can be used to solve them.

Addition Grid Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Subtraction Grid Facts

–	0	1	2	3	4	5	6	7	8	9	10
0	0–0										
1	1–0	1–1									
2	2–0	2–1	2–2								
3	3–0	3–1	3–2	3–3							
4	4–0	4–1	4–2	4–3	4–4						
5	5–0	5–1	5–2	5–3	5–4	5–5					
6	6–0	6–1	6–2	6–3	6–4	6–5	6–6				
7	7–0	7–1	7–2	7–3	7–4	7–5	7–6	7–7			
8	8–0	8–1	8–2	8–3	8–4	8–5	8–6	8–7	8–8		
9	9–0	9–1	9–2	9–3	9–4	9–5	9–6	9–7	9–8	9–9	
10	10–0	10–1	10–2	10–3	10–4	10–5	10–6	10–7	10–8	10–9	10–10
11		11–1	11–2	11–3	11–4	11–5	11–6	11–7	11–8	11–9	11–10
12			12–2	12–3	12–4	12–5	12–6	12–7	12–8	12–9	12–10
13				13–3	13–4	13–5	13–6	13–7	13–8	13–9	13–10
14					14–4	14–5	14–6	14–7	14–8	14–9	14–10
15						15–5	15–6	15–7	15–8	15–9	15–10
16							16–6	16–7	16–8	16–9	16–10
17								17–7	17–8	17–9	17–10
18									18–8	18–9	18–10
19										19–9	19–10
20											20–10

Calculation Strategies

At the heart of the programme are core addition and subtraction facts.

Essential facts (equivalent of times tables for addition and subtraction) - all future addition and subtraction calculations use these root addition and subtraction facts.

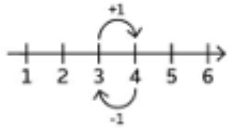
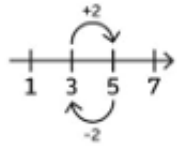
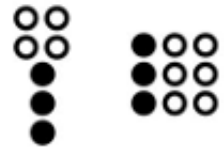
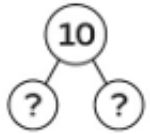
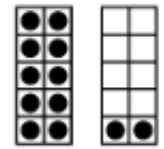
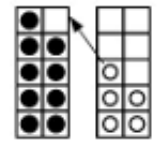
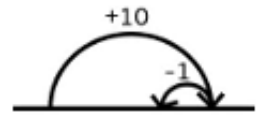
Number Sense Maths

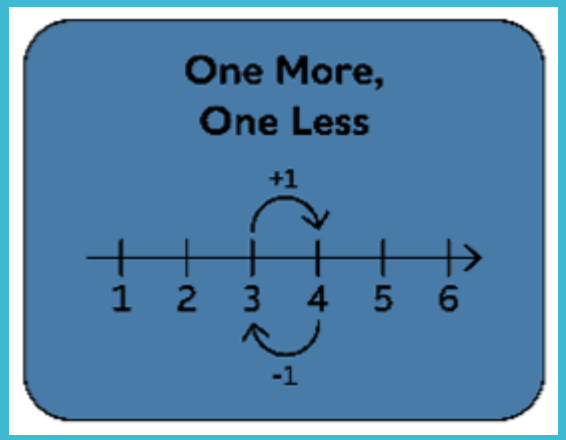
How do we teach the core facts?

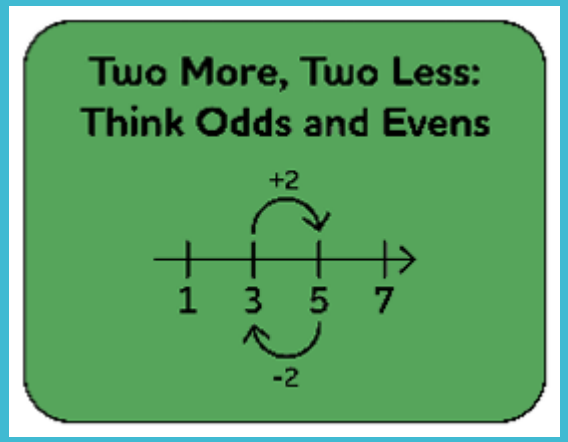
The core facts are taught alongside 12 calculation strategies.

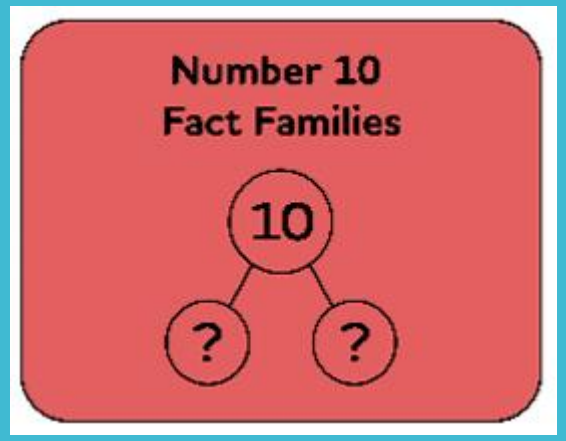
Learning and applying these strategies gives children a deep understanding of number and number relationships.

Explicit teaching of derived fact strategies is an effective route to fluency in addition and subtraction facts for all children.

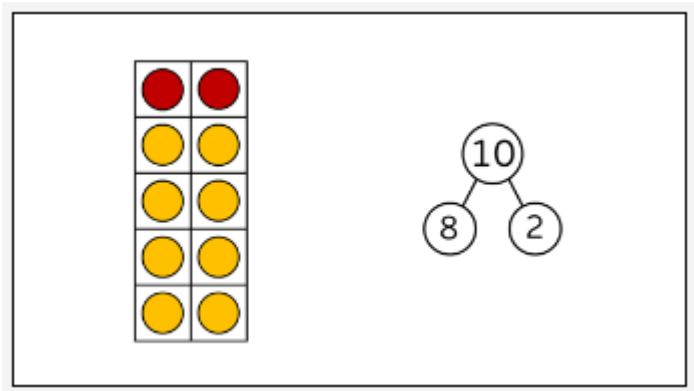
One More, One Less 	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).	**Number Neighbours: Spot the Difference** 	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2. Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)
Two More, Two Less: Think Odds and Evens 	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	**7 Tree and 9 Square** 	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.
Number 10 Fact Families 	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	**Ten and A Bit** 	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).
Five and A Bit 	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).	**Make Ten and Then...** 	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
Know about 0 **0**	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	**Adjust It** 	Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).
Doubles and Near Doubles 	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know 6 + 6 = 12 then 6 +7 and 5 + 7 is easy.	**Swap It**  1 + 6	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.

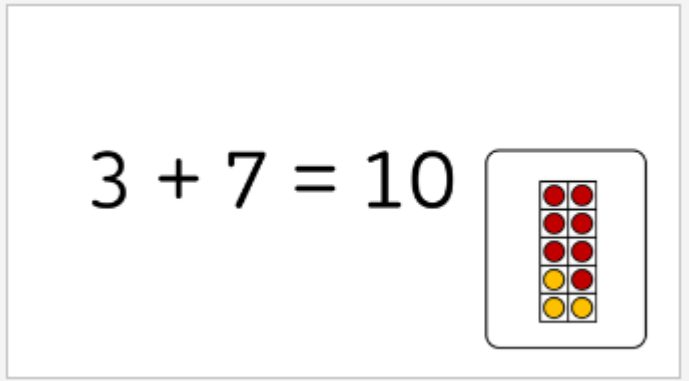

One More,
One Less
+1
1 2 3 4 5 6
-1

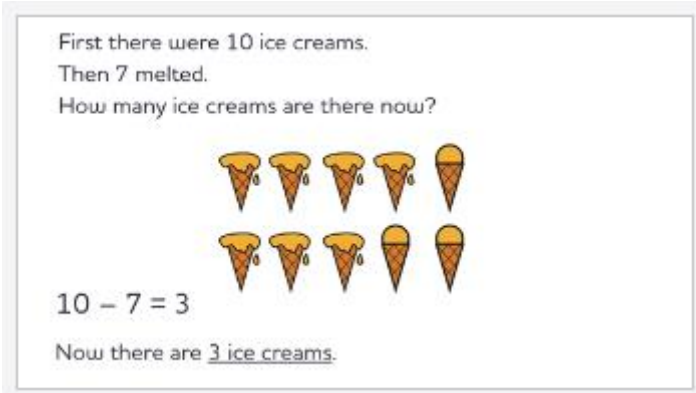

Two More, Two Less:
Think Odds and Evens
+2
1 3 5 7
-2

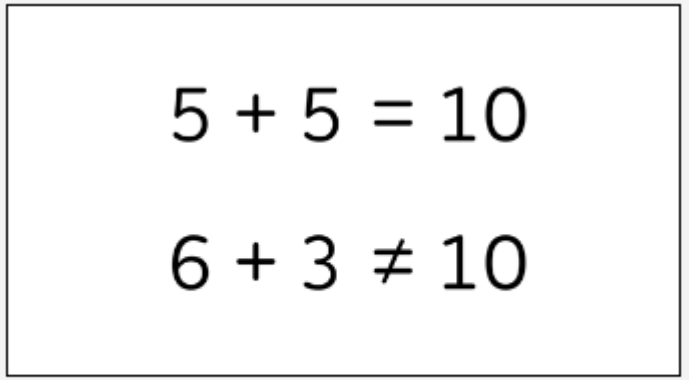


Different representations, story contexts and non-bonds.



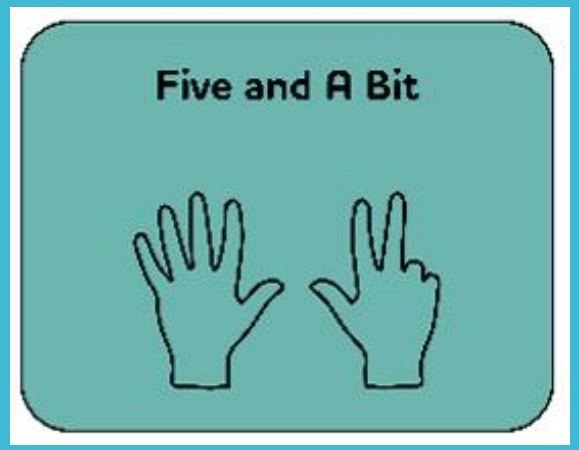






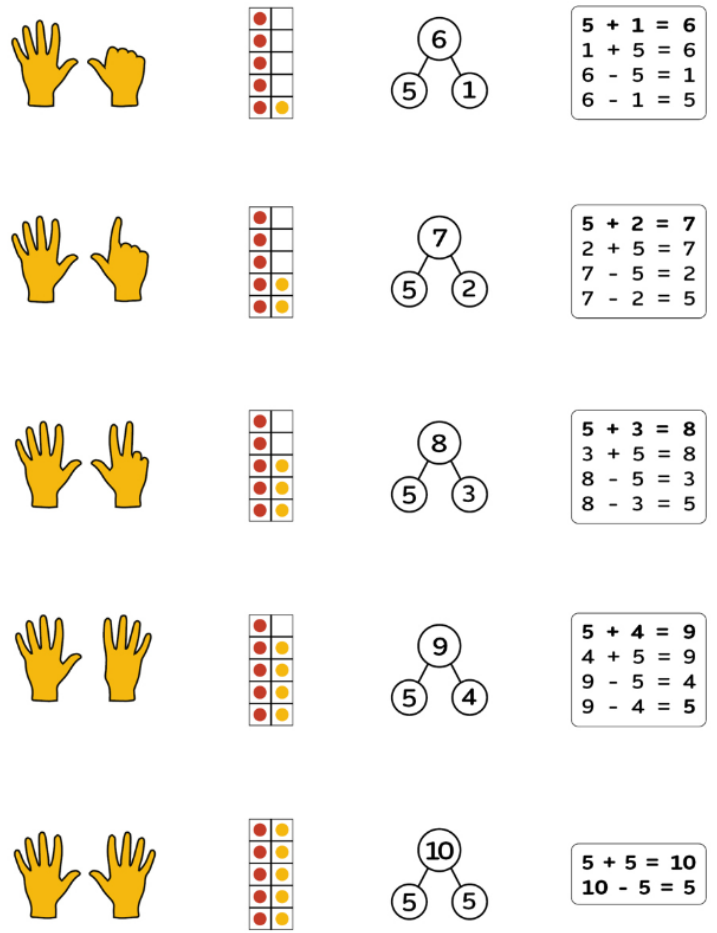
Does it make 10?

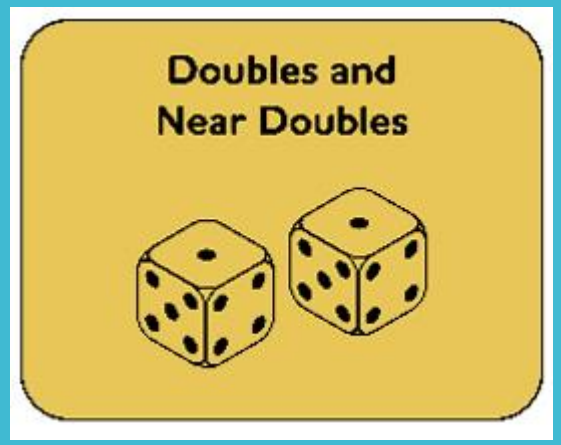
Knowledge of number bonds to 10 does not in itself, lead to fluency in the related addition and subtraction facts. Children must be explicitly taught, for example, that since we know that 2 and 8 make 10 we also know that 2+8=10, 8+2=10, 10-2=8, and 10-8 = 2, the fact family equations for the number pair 2 and 8.



This strategy teaches children to solve some of the trickier addition and subtraction facts within 10, such as 3+5 and 9-4. It builds on their understanding of the five and a bit structure of a number to its related addition and subtraction facts.

For example, we know 8 is made up of 5 and 3, and we can use this to solve 3+5, 5+3, 8-5 and 8-3.





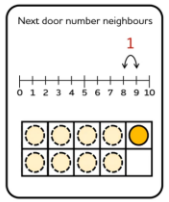
This concept that establishes that when we add or subtract 0 the starting number remains unchanged but addresses a common misconception that, for example 2+0=0 or 2-0=0 where a child thinks incorrectly that adding or subtracting 0 "gets rid" of what you had before

The strategy teaches children to learn doubles of the numbers 1 to 5 then how to use these double facts to solve related addition and subtraction calculations, for example, 8-4, and 4+4.

When the children are fluent in the doubles and halves number facts they are then taught how to use them to solve near doubles, for example, solving 5+4 as double 4 plus one more, or double 5 less one.

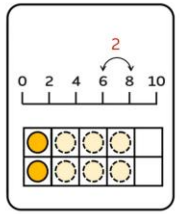
Children are taught to solve subtraction facts involving next door number neighbours, odd number neighbours and even number neighbours. When a subtraction involves next door number neighbours, such as 9-8, the difference is always 1. When a subtraction involves odd or even number neighbours, such as 8-6 or 7-5 the difference will always be 2.

9 - 8

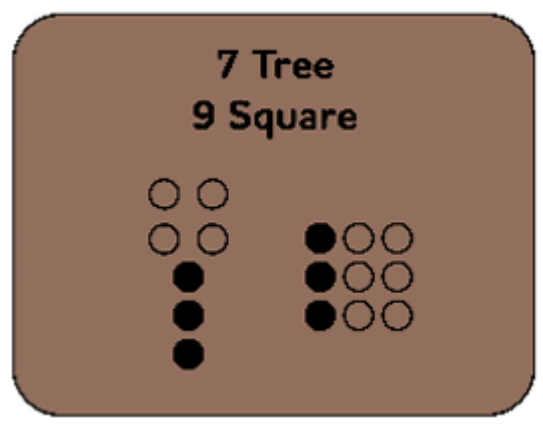


Fact 2 of 8

8 - 6



Fact 2 of 6

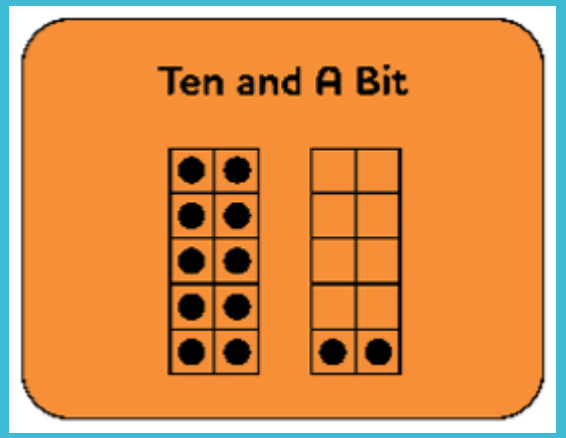


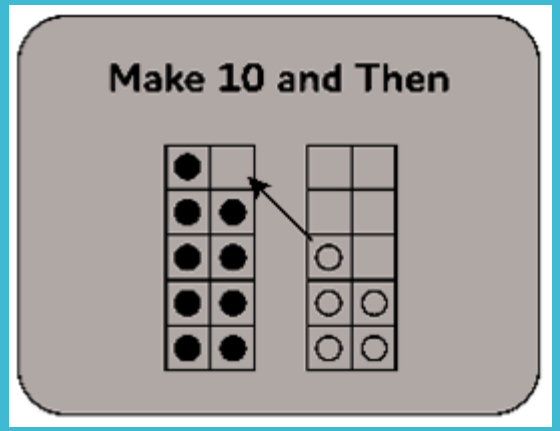
This strategy helps teach the children some of the trickiest addition and subtraction facts within 10 using two key visual images which will have been taught earlier in the programme.

The "7 tree" solves 3+4, 4+3, 7-4 and 7-3.

The "9 square" solves 3+6, 6+3, 9-6 and 9-3.

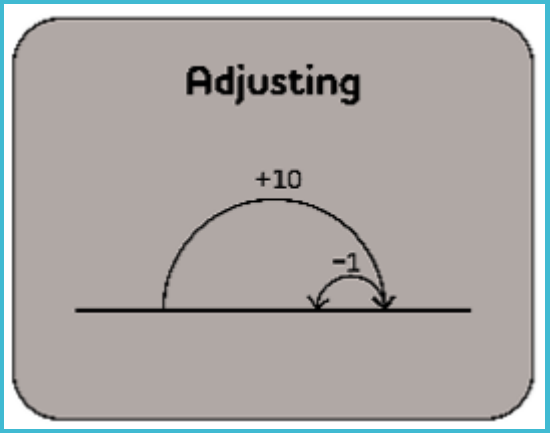
https://numbersensemaths.com/teacher-portal/nff/stages/stage-3/7-tree-and-9-square



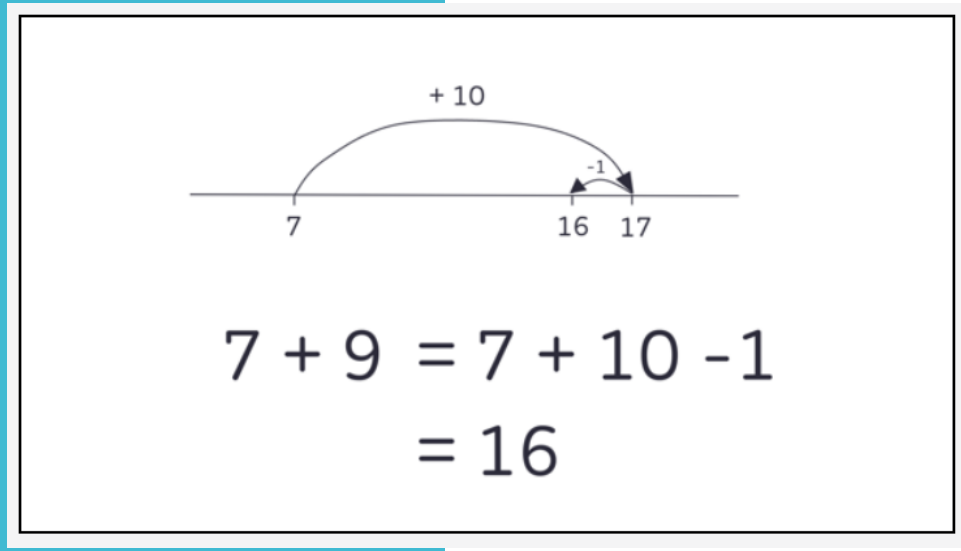


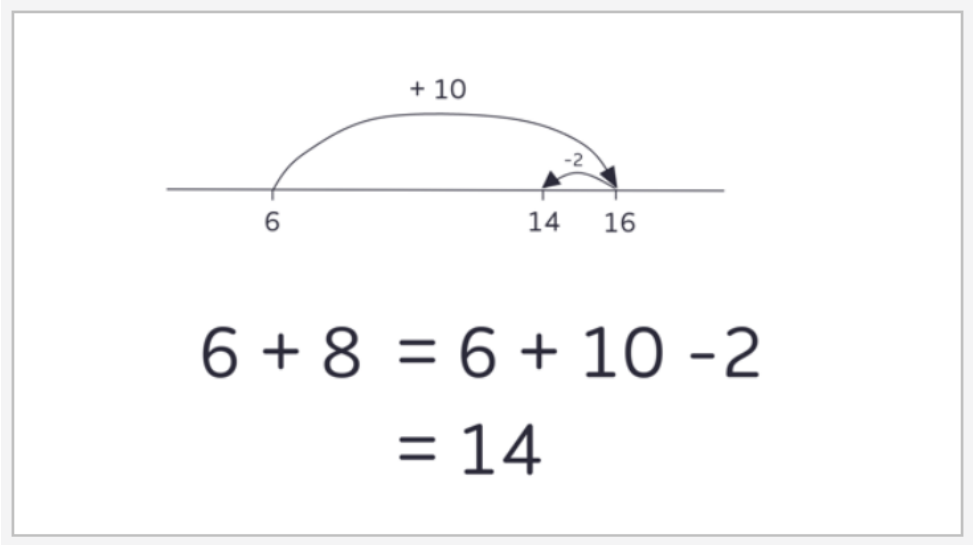
In Ten and a Bit, children are taught to recognise and visualise the numbers 11 to 20 on tens frames and then how to apply this knowledge to solve related addition and subtraction equations without counting. For example, 14 is made up of a 10 and a 4 and can be used to solve 10+4, 4+10, 14-10 and 14-4.

Becoming fluent in Make 10 and Then strategies for addition and subtraction will take a long time and lots of practice, but will provide strong foundations for children to move onto complex calculations like formal methods that bridge 10. This isn't about them simply being able to bridge 10, its about developing automaticity to do so and finding the most efficient, accurate and flexible method.



Children have been introduced to adjusting previously through the 'near doubles' strategy, and this strategy builds upon it by teaching two further types of adjusting which support adding and subtracting across 10: firstly, adding and subtracting 8 or 9 by adjusting from adding and subtracting 10, and secondly adjusting from known number bonds to 10 to add across 10, for example using 7 + 3 as a starting point to solve 7 + 4.





Strategies and facts

Addition Grid Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Subtraction Grid Facts

–	0	1	2	3	4	5	6	7	8	9	10
0	0–0										
1	1–0	1–1									
2	2–0	2–1	2–2								
3	3–0	3–1	3–2	3–3							
4	4–0	4–1	4–2	4–3	4–4						
5	5–0	5–1	5–2	5–3	5–4	5–5					
6	6–0	6–1	6–2	6–3	6–4	6–5	6–6				
7	7–0	7–1	7–2	7–3	7–4	7–5	7–6	7–7			
8	8–0	8–1	8–2	8–3	8–4	8–5	8–6	8–7	8–8		
9	9–0	9–1	9–2	9–3	9–4	9–5	9–6	9–7	9–8	9–9	
10	10–0	10–1	10–2	10–3	10–4	10–5	10–6	10–7	10–8	10–9	10–10
11		11–1	11–2	11–3	11–4	11–5	11–6	11–7	11–8	11–9	11–10
12			12–2	12–3	12–4	12–5	12–6	12–7	12–8	12–9	12–10
13				13–3	13–4	13–5	13–6	13–7	13–8	13–9	13–10
14					14–4	14–5	14–6	14–7	14–8	14–9	14–10
15						15–5	15–6	15–7	15–8	15–9	15–10
16							16–6	16–7	16–8	16–9	16–10
17								17–7	17–8	17–9	17–10
18									18–8	18–9	18–10
19										19–9	19–10
20											20–10

Number Sense Maths

Calculation Strategies

Coming back to the grid facts, we can see how each of the different strategies that the programme teaches, maps to the core facts.

Play in pairs
Ace – 10 (+ JQK)

Classroom Fluency with a pack of cards

GAME 1 (could remove numbers 6, 7, 8, 9 for KS1):

Deal the top card.

Deal the next card. Tell your partner the strategy you will use to add the cards: next door neighbours? Doubles? Near doubles? Bonds to 10?

Deal the next card. Partner decides strategy to add. Repeat.

GAME 2 (KS2 speed game):

Turn over the first card. Take it in turns to turn over the next card, adding as you go. First pair to finish stand up.

GAME 3 (KS2 speed game):

Start at 55, turn over the first card and subtract number, take turns to turn over cards subtracting as you go. First pair to get to 0.

GAME 4 (KS2)

Add in J, Q, K. Choose a number (which will become your number to multiply each card by). Turn over cards and take turns to multiply the card by chosen number.

Play in pairs
Full deck

- **Game 5** (KS2)

Start at 340. Start with deck face down. Turn over a card and subtract from value from 340. Next player turn over next card and subtract from next total. When all cards have been turned over they should end up on zero (unless a mistake has been made!)

*adapt for KS1 – use all suits A-5 – total 60

Game 6 (KS2)

Start on 0. Start with deck face down. Turn a card over. Turn the second card over and add to the first card. Continue adding to the cumulative total. When all cards have been turned over and added, the total should be 340 (unless a mistake has been made!)

*adapt for KS1 – use all suits A-5 – total 60

Outdoor games

Game 7 – relay

Use a 2x full suit Ace – K cards per team

Shuffle deck and place at one end of playing area (hall, playground).

Player 1 runs to deck, draws two cards and performs specified operation and records correct answer before running back and tagging next member of team to repeat process.

Team whose players complete relay first win